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Title: Neutron Electric Dipole Moment

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Neutron Electric Dipole Moment

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Introduction

Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar `arXiv:hep-ph/0008248`.

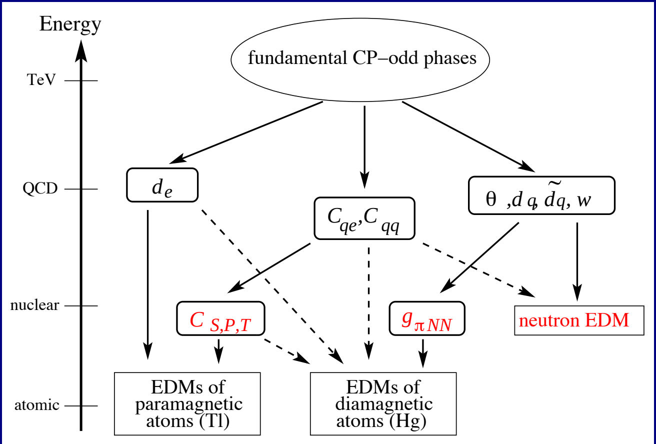
- CP-violating mass term and effective $\Theta G\tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

Introduction
Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\mathrm{EW}}/M^2_{\mathrm{BSM}}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\mathrm{BSM}}$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
Form Factors

Vector form-factors
Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\begin{aligned}\langle N|V_\mu(q)|N\rangle &= \overline{u}_N \Bigg[\gamma_\mu\, F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu\, \frac{F_2(q^2)}{2m_N} \\ &\qquad + (2i\, m_N \gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\, \frac{F_A(q^2)}{m_N^2} \\ &\qquad + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5\, \frac{F_3(q^2)}{2m_N}\Bigg] u_N\end{aligned}$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A and F_3 violate P; F_3 violates CP.

Dirac Equation

Free neutrons

By Lorentz invariance, an on-shell spin-$\frac{1}{2}$ massive field ψ obeys

$$(\text{`}e^{\beta\gamma_5}\text{'}\not{p} + e^{i\alpha\gamma_5} m)\psi = 0\,.$$

This 'free' equation has space-time discrete symmetries:

$$\begin{aligned}
\mathcal{P}: \quad & \psi(\vec{x},t) \to e^{(\beta - i\alpha)\gamma_5}\,\gamma_0\,\psi(-\vec{x},t)\\
\mathcal{C}: \quad & \psi(\vec{x},t) \to i\,e^{\beta\gamma_5}\,\gamma_2\,\psi^*(\vec{x},t)\\
\mathcal{T}: \quad & \psi(\vec{x},t) \to -\,e^{-i\alpha\gamma_5}\,\gamma_1\gamma_3\,\psi^*(-\vec{x},t)
\end{aligned}$$

- Even when theory does not have these symmetries,
- asymptotic states always do,
- but operators have extra γ_5 phases.
- $e^{(-\beta+i\alpha)\gamma_5/2}\,\psi$ has standard phases.

Dirac Equation
Phase conventions

Consider $N = (\bar{u}^c \gamma_5 d) u$ with usual phases for u and d.
If theory has $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, N has the same phases.
So, in a symmetric theory, $\alpha = \beta = 0$.
Otherwise, by Lorentz invariance, we still have

$$\mathcal{P} = \left[e^{\beta(p^2)\gamma_5} \Pi(p^2) \not{p} - e^{i\alpha(p^2)\gamma_5} \Sigma(p^2) m \right]^{-1} .$$

Like the free Dirac equation, but $e^{(\beta(p^2) - i\alpha(p^2))\gamma_5}$ is not local.

Dirac Equation

State-dependent phases

By Källen-Lehman spectral representation, the propagator is

$$\sum \rho(\mu^2) Z_N(\mu^2) \frac{e^{-\beta(\mu^2)\gamma_5} \not{p} + e^{-i\alpha(\mu^2)\gamma_5} Z_m(\mu^2)\mu}{p^2 - (Z_m(\mu^2)\mu)^2} \, .$$

When there is no overall symmetry operator, phases state-dependent.

$N_{\rm st} \equiv e^{(-\beta(m_N^2)+i\alpha(m_N^2))\gamma_5/2} N$ has standard transformations and equation of motion, but only for the neutron; the excited states have non-standard phases.

Dirac Equation

Electric Dipole Moment

$e\sigma \cdot B$ is even under $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$,
$e\sigma \cdot E$ is odd under $\mathcal{P}$ and $\mathcal{T}$.

$$\Sigma \cdot F \propto \begin{pmatrix} \sigma \cdot B & i\sigma \cdot E \\ i\sigma \cdot E & \sigma \cdot B \end{pmatrix},$$

which is $\sigma \cdot B$ in the rest frame iff $\not{p} \pm m = 0$.
In general, we need to use $e^{i\alpha\gamma_5}\Sigma \cdot F$.
So, important to use $N_{\rm st}$ instead of N in analyses.
At the Green's function level, this is

$$\langle T N_{\rm st} O \bar{N}_{\rm st} \rangle = e^{(-\beta(m_N^2)+i\alpha(m_N^2))\gamma_5/2} \langle T N O \bar{N} \rangle e^{(\beta(m_N^2)+i\alpha(m_N^2))\gamma_5/2}.$$

Abramczyk, Aoki, Blum, Izubichi, Ohki, and Syritsyn arXiv:1701.07792 [hep-lat]

Lattice Calculation

Methods

Two methods for calculating the EDM.

1. Spin dependent energy in an external electromagnetic field.
    - Need to include background EM fields in gauge generation.
    - Can ‘reweight’ with disconnected loops.
    - EM flux quantized: so large fields in small volumes.
    - Need derivative at zero.
2. F_3 Form factor
    - Need to calculate at non-zero momentum transfer.
    - Need derivative at zero-momentum: difficult on small lattices.
    - Need disconnected insertion of currents.
    - Need to account for γ_5 phase.
    - Many operators need 4-point functions, analytic continuation, or source methods.

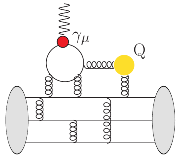
Lattice Calculation

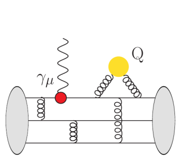
Topological charge and Weinberg operator

To find the contribution of $\bar{\Theta}$, we need the correlation between the electric current and the topological charge. Similar calculation for the Weinberg operator.

$$\left\langle n \left| (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d) Q \right| n \right\rangle \quad = $$

$$\frac{1}{2}\left\langle n \left| (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) Q \right| n \right\rangle \quad + \quad \frac{1}{6}\left\langle n \left| (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) Q \right| n \right\rangle$$





See also Liu, Liang, Yang, arXiv:1705.06358

Probably no non-zero signal yet.

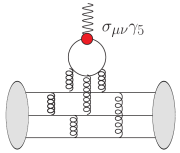
- Shintani *et al.*, Physical Review **D72** (2005) 014504.
- Berruto *et al.*, Physical Review **D73** (2006) 054509.
- Guo *et al.*, Physical Review Letters **115** (2015) 062001.
- Shindler *et al.*, Physical Review **D92** (2015) 094518.
- Alexandrou *et al.*, Physical Review **D93** (2016) 074503.
- Shintani *et al.*, Physica Review **D93** 094503.

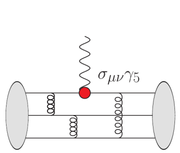
need to be reanalyzed.
Abramczyk estimate that $F_3 \lesssim 0.07\bar{\theta}$ at 1σ.

Lattice Calculation
Quark Electric Dipole Moment

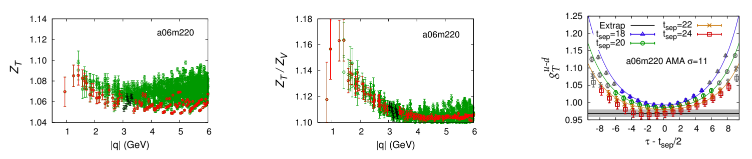
Since the quark electric dipole moment directly couples to the electric field, we just need to calculate its matrix elements in the neutron state.

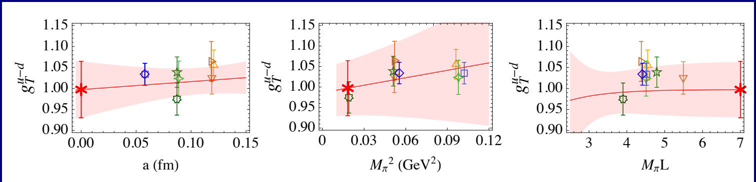
$$\left\langle n \left| d_u^\gamma \, \bar{u}\sigma^{\mu\nu} u + d_d^\gamma \, \bar{d}\sigma^{\mu\nu} d \right| \right\rangle =$$
$$\frac{d_u^\gamma + d_d^\gamma}{2} \left\langle n \left| \bar{u}\sigma^{\mu\nu} u + \bar{d}\sigma^{\mu\nu} d \right| n \right\rangle + \frac{d_u^\gamma - d_d^\gamma}{2} \left\langle n \left| \bar{u}\sigma^{\mu\nu} u - \bar{d}\sigma^{\mu\nu} d \right| n \right\rangle$$



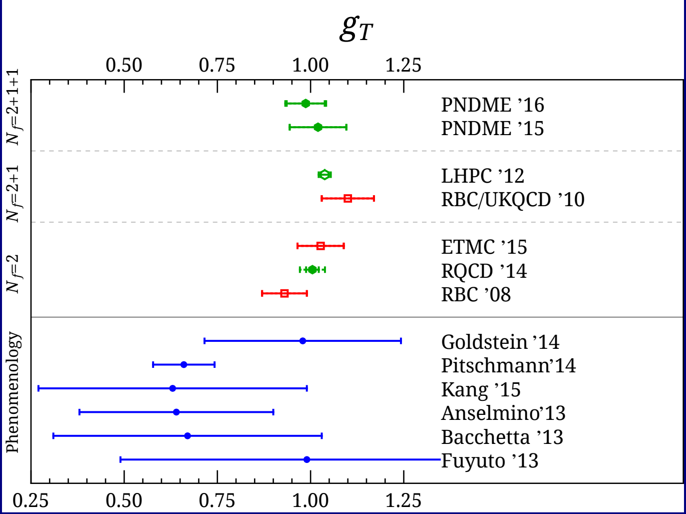


Parity mixing higher order in α_{EW}: so, result is same as tensor charge.





Results for $u + d$ similar. $g_T^u = 0.792(14)$; $g_T^d = -0.194(14)$.
Disconnected contribution small.

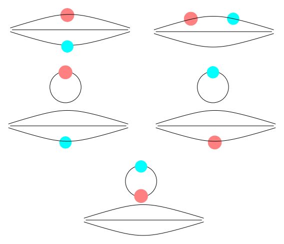

g_T
0.50
0.75
1.00
1.25
N_f=2+1+1
PNDME '16
PNDME '15
N_f=2+1
LHPC '12
RBC/UKQCD '10
N_f=2
ETMC '15
RQCD '14
RBC '08
Phenomenology
Goldstein '14
Pitschmann'14
Kang '15
Anselmino'13
Bacchetta '13
Fuyuto '13
0.25
0.50
0.75
1.00
1.25

Lattice Calculation

Quark Chromoelectric Moment

nEDM from quark chromoelectric moment is a four-point function:

$$\left\langle n \left| (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d) \int d^4x \, (d_u^G \, \bar{u}\sigma^{\nu\kappa}u + d_d^G \, \bar{d}\sigma^{\nu\kappa}d) \, \tilde{G}_{\nu\kappa} \right| n \right\rangle$$



Lattice Calculation

Reduction to three-point function

The quark chromo-EDM operator is a quark bilinear.
Schwinger source method: Add it to the Dirac operator in the propagator inversion routine:

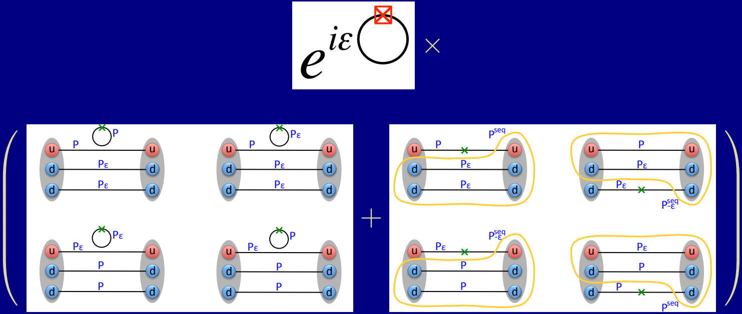
$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

The fermion determinant gives a 'reweighting factor'

$$\begin{aligned}
&\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})} \\
&\quad = \quad \exp \mathrm{Tr} \ln \left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] \\
&\quad \approx \quad \exp\left[i\epsilon\,\mathrm{Tr}\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right].
\end{aligned}$$

Lattice Calculation

Chromo-edm diagrams

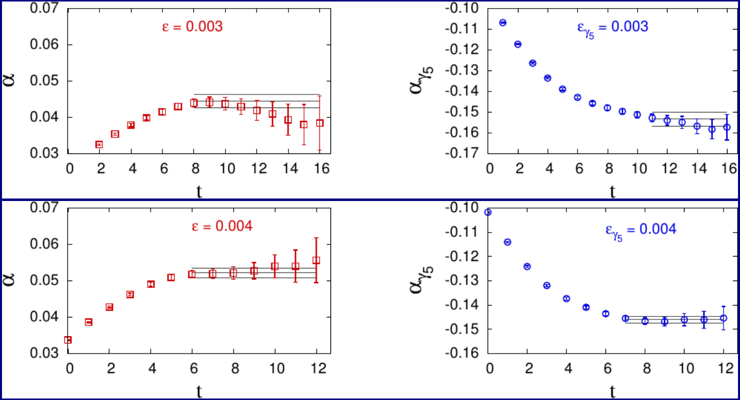


The chromoEDM operator is dimension 5.

Uncontrolled divergences unless $\epsilon \lesssim 4\pi a \Lambda_{\rm QCD} \sim 1$.

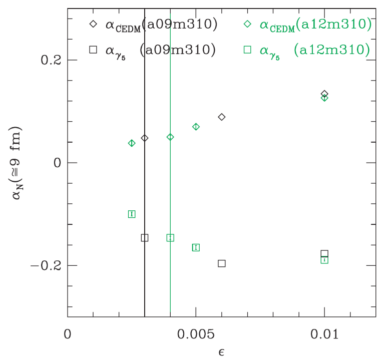
Need to check linearity.

Two point functions
Neutron Propagator



Preliminary; Connected Diagrams Only

Two point functions
Linearity



Preliminary; Connected Diagrams Only

Use $\epsilon \approx \frac{a}{30\text{fm}} \approx 6.6\text{MeV}\, a \approx 0.36\, ma$ for experiments.

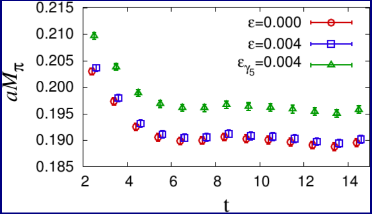
Two point functions

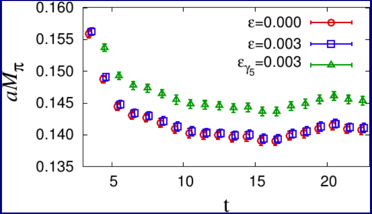
Connected γ_5

$$a(\not{D}+m)+i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha_q\gamma_5}\left(a\not{D}+am_\epsilon\right)e^{\frac{i}{2}\alpha_q\gamma_5}$$

where $\alpha_q \equiv \tan^{-1}\frac{\epsilon}{am}$

and $am_\epsilon \equiv \sqrt{(am)^2+\epsilon^2}$





	a12m310	a09m310
$am^0 \equiv \frac{1}{2\kappa} - 4$	-0.0695	-0.05138
$am_{cr} \equiv \frac{1}{2\kappa_c} - 4$	-0.08058	-0.05943
$am \equiv am^0 - am_{cr}$	0.01108	0.00805
ϵ	0.004	0.003
am_ϵ	0.01178	0.00859
M_π^0	0.1900(4)	0.1404(3)
M_π^{CEDM}	0.1906(4)	0.1407(3)
$M_\pi^{\gamma_5}$	0.1961(4)	0.1450 (3)
$M_\pi^0 \times \sqrt{\frac{m_\epsilon}{m}}$	0.1959(4)	0.1450(3)

Three point functions

F_3 Form factor from CEDM

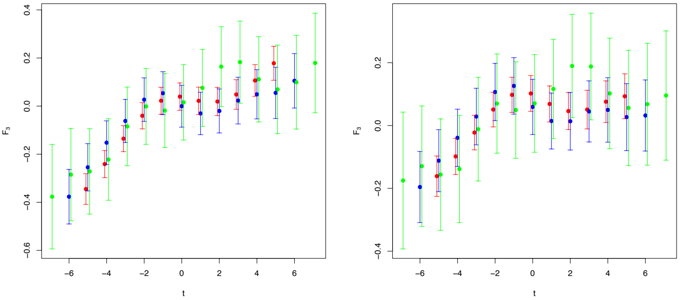


Figure 4: Signal in F_3 from the insertion of the cEDM operator in the u (left) and d (right) quarks

Preliminary; Connected Diagrams Only

Three point functions
Renormalization

RI-SMOM scheme for non-perturbative renormalization worked out.
Most divergent mixing with $a^{-2}\alpha_s\bar{\psi}\gamma_5\psi$ even with chiral symmetry.
Effect same as of $(\alpha_s/ma^2)G\cdot\tilde{G}$.
Current estimates of nEDM:

- CEDM is O(1).
- $(\alpha_s/ma^2)G\cdot\tilde{G}$ contribution is O(1)-O(10) at $a\sim 0.1$fm

Not present in connected diagrams with good chiral symmetry!
For Wilson fermions, $O(a)$ chiral breaking gives multiplicative correction.